HE 215 : Nuclear & Particle Physics Course

Dr. Jyothsna Rani Komaragiri

Centre for High Energy Physics, IISc

Webpage: `http://chep.iisc.ac.in/Personnel/jyothsna.html`

e-mail:
jyothsna@iisc.ac.in,
jyothsna.komaragiri@gmail.com

September 2018 Lectures

Outline I

- Symmetries, Groups and Conservation Laws
  - Noether's Theorem
  - Spin and Orbital Angular Momentum
  - Clebsch-Gordon Coefficients
  - Spin 1/2
  - Isospin
  - Parity
  - Charge Conjugation
  - G-Parity
  - CP Symmetry

Symmetries, Groups and Conservation Laws

This is chapter 4 in Griffiths.

Symmetries, Groups and Conservation Laws

You cannot study the Standard Model without studying symmetries. Group theory is interwoven in all discussions of modern particle physics.

You need not panic when you see:

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

So we will briefly discuss group theory, continuous and discrete symmetries and resulting conservation laws.

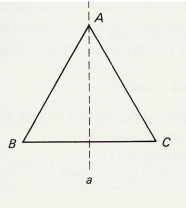
Symmetries

What exactly is a symmetry?

Symmetry

A symmetry is an operation you can perform on a system which leaves it invariant.

Consider an equilateral triangle:



It is invariant under a clockwise rotation of 120° or by a counterclockwise rotation by 120°, by flipping about Aa or similar axes through B or C (or by doing nothing at all).

Symmetries

Consider an odd function

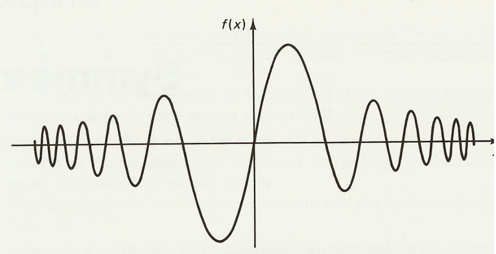


Figure 4.1 An odd function.

It is symmetric under a substititon of $f(x) \rightarrow -f(-x)$. Even if the function itself is unknown, the symmetry alone tells gives you information about the theory it describes and can be a practically useful.

Noether's Theorem

Each symmetry of nature yields a conservation law; conversely, every conservation law yields an underlying symmetry

Symmetry	Conservation Law
Translation in time	Energy
Translation in space	Momentum
Rotation	Angular momentum
Gauge transformation	Charge

- Some symmetries are only approximate. Still useful.
  Example: observation of neutrino oscillations means that Lepton flavour conservation is not exact.
- Sometimes we find conservation laws that do not correspond to any previously known symmetry. Could be something to puzzle theorists, could be only an approximate symmetry.
  Example: Lepton and Baryon number conservation are not associated with fundamental symmetries.

Translation - Momentum

Consider an isolated non-relativistic system consisting of 2 particles interacting through a potential that depends on their relative separation. The kinetic and potential energies are:

$$\begin{aligned} T &= \frac{1}{2}m_1\dot{\boldsymbol{r}}_1^2 + \frac{1}{2}m_2\dot{\boldsymbol{r}}_2^2 \\ V &= V(\boldsymbol{r}_1 - \boldsymbol{r}_2) \end{aligned}$$

where $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are their coordinates measures relative to some origin. The dynamical equations become

$$\begin{aligned} m_1\ddot{\boldsymbol{r}}_1 &= -\nabla_1 V(\boldsymbol{r}_1 - \boldsymbol{r}_2) = -\frac{\partial}{\partial \boldsymbol{r}_1}V(\boldsymbol{r}_1 - \boldsymbol{r}_2) \\ m_2\ddot{\boldsymbol{r}}_2 &= -\nabla_2 V(\boldsymbol{r}_1 - \boldsymbol{r}_2) = -\frac{\partial}{\partial \boldsymbol{r}_2}V(\boldsymbol{r}_1 - \boldsymbol{r}_2) \end{aligned}$$

Translation - Momentum

Now translate the origin by a constant vector $\boldsymbol{a}$

$$\begin{aligned} \boldsymbol{r}_1 &\rightarrow \boldsymbol{r}_1' = \boldsymbol{r}_1 + \boldsymbol{a} \\ \boldsymbol{r}_2 &\rightarrow \boldsymbol{r}_2' = \boldsymbol{r}_2 + \boldsymbol{a} \end{aligned}$$

The dynamical equations do not change since

$$V(\boldsymbol{r}_1 - \boldsymbol{r}_2) \rightarrow V(\boldsymbol{r}_1 + \boldsymbol{a} - \boldsymbol{r}_2 - \boldsymbol{a}) = V(\boldsymbol{r}_1 - \boldsymbol{r}_2)$$

So, the system is invariant under a spatial translation.

Translation - Momentum

Let's look at the total force on the system:

$$\boldsymbol{F}_{TOT} = \boldsymbol{F}_1 + \boldsymbol{F}_2 = -\nabla_1 V(\boldsymbol{r}_1 - \boldsymbol{r}_2) - \nabla_2 V(\boldsymbol{r}_1 - \boldsymbol{r}_2) = 0$$

(the form of the potential tell us that the force on one is the same magnitude and opposite direction of the force on the other).

$$\frac{d\boldsymbol{P}_{TOT}}{dt} = \boldsymbol{F}_{TOT} = 0$$

All this tells us really is that, with this form of the potential for a 2-body system, the force equations are invariant under translation and momentum is conserved. However, we could in this way build a Lagrangian and get the same result in a more general case.

QM Example: Translation - Momentum

- Translate the wavefunction $\psi(x)$ by an amount a: $\psi(x) \longrightarrow \psi(x+a)$
- Now expand $\psi(x+a)$ as a Taylor series about $\psi(x)$

$$\begin{aligned}
\psi(x+a) &= \psi(x) + a\,\frac{\partial\psi}{\partial x}\bigg|_x + \frac{a^2}{2!}\,\frac{\partial^2\psi}{\partial x^2}\bigg|_x + \ldots \\
&= \left(\sum_{n=0}^{\infty}\frac{a^n}{n!}\,\frac{\partial^n}{\partial x^n}\right)\psi(x) = U(a)\psi(x)
\end{aligned}$$

where $\quad U(a) = \exp\left[a\,\frac{\partial}{\partial x}\right]$

- If our physical system is indeed invariant under translations, then

$$\begin{aligned}
\langle\psi(x)|\psi(x)\rangle &= \langle\psi(x+a)|\psi(x+a)\rangle \\
&= \langle U(a)\psi(x)|U(a)\psi(x)\rangle \\
&= \left\langle\psi(x)|U^\dagger(a)U(a)\psi(x)\right\rangle
\end{aligned}$$

then, $U^\dagger U = 1$, or in other words $U^\dagger = U^{-1}$ (unitary matrix).

Connection to Hermitian Operators

- Recall that in Quantum Mechanics, every physical observable is represented by a Hermitian operator $(H^\dagger = H)$.
- Every Hermitian operator has the form $U = e^{iH}$.
- Factoring out a couple of constants so that $U(a) = \exp\left[iHa/\hbar\right]$ comparing to our previous result of $U(a) = \exp\left[a\,\partial/\partial x\right]$, we find

$$p = \frac{\hbar}{i}\frac{\partial}{\partial x}$$

is a Hermitian operator which generates spatial translations.

Group Theory

Group theory is the mathematical description of symmetries, namely operations which leave a system invariant.

A set of symmetry operations on any system must have the following properties:

1. Closure: the product $R_i R_j$ is another member of the set R_k
2. Identity: There is an element I such that $IR_i = R_i I = R_i$ for all R_i
3. Inverse: for every element R_i there is an inverse R_i^{-1} such that $R_i R_i^{-1} = R_i^{-1} R_i = I$
4. Associativity: $R_i(R_j R_k) = (R_i R_j) R_k$

Notice that the operations need not commute. If they do they are called Abelian, if they do not then they are non-Abelian.
We are dealing with groups with continuously connected elements: Lie groups.

Group Theory

Every group can be represented by a group of matrices.

Important groups in particle physics

U(n)	unitary ($\tilde{U}^* U = 1$)
SU(n)	special unitary with determinant = 1
O(n)	orthogonal ($\tilde{O} O = 1$)
SO(n)	special orthogonal with determinant = 1

Spin and Orbital Angular Momentum

It is time for a quick QM reminder about spin and orbital angular momentum. Remember that every particle has an **intrinsic property** called spin in addition to the angular momentum associated with its motion.

In QM it is impossible to measure all components of the angular momentum vector $\boldsymbol{L}$ simultaneously. We generally discuss the squared magnitude of the vector and the z-component:

$$L^2, L_z$$

There are certain allowed values of L^2

$$l(l+1)\hbar^2$$

where l is a non-negative integer

$$l = 0, 1, 2, 3, \ldots$$

Spin and Orbital Angular Momentum

For a given value of l, a measurement of L_z yields

$$m_l\hbar$$

where m_l is

$$m_l = -l, -l+1, -l+2, \ldots, -1, 0, 1, 2, \ldots, l-1, l$$

with $2l+1$ possibilities.

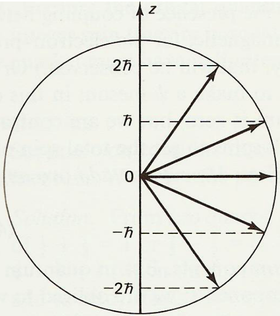


Figure 4.3 Possible orientations of the angular momentum vector for $l = 2$.

Spin and Orbital Angular Momentum

Similar relationships exist for spin angular momentum. A measurement of S^2 can only return values of the form

$$s(s+1)\hbar^2$$

Though the spin can have half-integer values

$$s = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots$$

A measurement of S_z must yield an answer of the form

$$m_s\hbar$$

where

$$m_s = -s, -s+1, \ldots, s-1, s$$

with $2s+1$ possibilities.
Each particle has a specific value of s which is an intrinsic property of the particle type. Half-integers are fermions and integers are bosons.

Addition of Angular Momenta

Angular momentum states are labelled with a ket

$$|lm_l\rangle \;\; or \;\; |sm_s\rangle$$

An electron occupying the orbital state $|3-1\rangle$ and the spin state $\left|\frac{1}{2}\frac{1}{2}\right\rangle$ has

$$\begin{aligned} l &= 3 \\ m_l &= -1 \\ s &= \frac{1}{2} \\ m_s &= \frac{1}{2} \end{aligned}$$

We are most interested in the total angular momentum (orbital + spin):

$$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$$

Addition of Angular Momenta

How do we add these angular momenta (ie. $\boldsymbol{J} = \boldsymbol{J}_1 + \boldsymbol{J}_2$)?

The z-components simply add

$$m = m_1 + m_2$$

However, the magnitudes do not add so simply. We get:

$$j = |j_1 - j_2|, |j_1 - j_2| + 1, \ldots, (j_1 + j_2) - 1, (j_1 + j_2)$$

i.e.

$$|j_1 - j_2| \leq j \leq j_1 + j_2$$

Example: a particle of spin 1 in an orbital state $l = 3$ could have total angular momentum:
$j = 4$ $(J^2 = 20)$, or $j = 3$ $(J^2 = 12)$, or $j = 2$ $(J^2 = 6)$ with $\hbar = 1$

Example 4.1

Example 4.1

A quark and an anti-quark are bound together, in a state of zero orbital angular momentum, to form a meson. What are the possible values of the meson spin?

Quarks are spin=1/2 so we can either have spin 1 or spin 0.

Spin-0 gives 'pseudo scalar' mesons π's, K's, η's, etc.
Spin-1 gives 'vector' mesons ρ's, K^*'s, ϕ, ω 'psuedo' will be explained later in the lectures.

Example 4.2

Example 4.2

Suppose you combine three quarks in a state of zero orbital angular momentum. What are the possible spins of the resulting baryon?

First add two quarks of spin-1/2 and get either 1 or 0. Then add another spin-1/2. So, we can have

$$\begin{aligned} 1+\frac{1}{2} &= \frac{3}{2} \\ 1-\frac{1}{2} &= \frac{1}{2} \\ 0+\frac{1}{2} &= \frac{1}{2} \end{aligned}$$

Fermions vs Bosons

- Particles with integer spin are bosons and obey Bose-Einstein statistics (i.e., symmetric w.r.t. exchange of identical particles).
- Particles with half-integer spin are fermions and obey Fermi-Dirac statistics (i.e., antisymmetric w.r.t. exchange of identical particles).
- Interchanging two particles is equivalent to a 2π relative rotation
  The unitary transformation which effects rotations is

$$U(\theta) = \exp\left[\frac{i}{\hbar}\mathbf{J}\cdot\theta\right]$$

- particles with integer spin, $U(2\pi) = \exp(2n\pi i) = 1 \Rightarrow$ bosons.
- particles with half-integer spin, $U(2\pi) = \exp[2(n+1/2)\pi i] = -1 \Rightarrow$ fermions.
- For a 2-particle system, $\psi(12) = \pm\psi(12)$

Clebsch-Gordon Coefficients

You may wish to explicitely decompose a system. Consider a two particle system $|sm\rangle$ in terms of the two individual particles:

$$|sm\rangle = \sum_{m_1+m_2=m} C^{s_1 s_2 s}_{m_1 m_2 m} |s_1 m_1\rangle |s_2 m_2\rangle$$

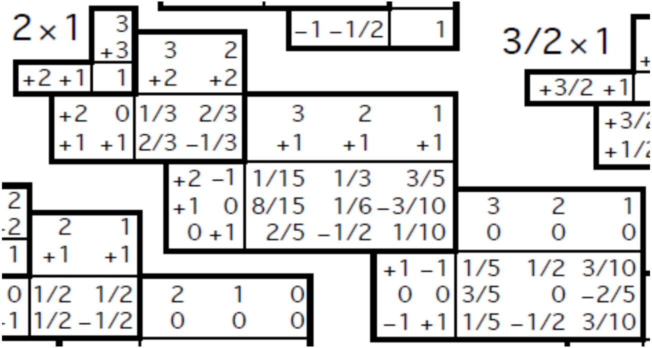
where the constants $C^{s_1 s_2 s}_{m_1 m_2 m}$ are the Clebsch-Gordon coefficients. You can use this to (for example) decompose the $|30\rangle$ state into

$$|30\rangle = \frac{1}{\sqrt{5}} |21\rangle |1-1\rangle + \sqrt{\frac{3}{5}} |20\rangle |10\rangle + \frac{1}{\sqrt{5}} |2-1\rangle |11\rangle$$

If 2 particles, of spin 2 and spin 1, are at rest in a box and the total spin is 3 and its z component is 0, then a measurement of S_z could return

$$\begin{array}{cl} \hbar & \text{with probability } 1/5 \\ 0 & \text{with probability } 3/5 \\ -\hbar & \text{with probability } 1/5 \end{array}$$

Notice that the probabilities sum to 1.



Look for column labeled 3 0

$$|30\rangle = \frac{1}{\sqrt{5}}\,|21\rangle\,|1-1\rangle + \sqrt{\frac{3}{5}}\,|20\rangle\,|10\rangle + \frac{1}{\sqrt{5}}\,|2-1\rangle\,|11\rangle$$

Clebsch-Gordon Coefficients

Griffiths actually does it with total angular momentum and starts the other way around:

$$|j_1 m_1\rangle\,|j_2 m_2\rangle = \sum_{j=|j_1-j_2|}^{j_1+j_2} C^{j j_1 j_2}_{m m_1 m_2}\,|jm\rangle\,,\, m = m_1 + m_2$$

Clebsch-Gordon Coefficients

Example 4.3

The electron in a hydrogen atom occupies the orbital state $|2\,-1\rangle$ and the spin state $\left|\frac{1}{2}\,\frac{1}{2}\right\rangle$. If we measure J^2, what values might we get and what is the probability of each?

The z components just add: $m = -1 + \frac{1}{2} = -\frac{1}{2}$
The possible values of j are: $j = l + s = 2 + \frac{1}{2} = \frac{5}{2}$ or
$j = l - s = 2 - \frac{1}{2} = \frac{3}{2}$
We then need to look at the Clebsch-Gordon table for:
$j_1 = 2$ and $j_2 = 1/2$

Clebsch-Gordon Coefficients

		5/2									
$2 \times 1/2$		+5/2	5/2	3/2							
+2	1/2	1	3/2	+3/2							
	+2	−1/2	1/5	4/5	5/2	3/2					
	+1	+1/2	4/5	−1/5	+1/2	+1/2					
			1	−1/2	2/5	3/5	5/2	3/2			
			0	+1/2	3/5	−2/5	−1/2	−1/2			
					0	−1/2	3/5	2/5	5/2	3/2	
					−1	+1/2	2/5	−3/5	−3/2	−3/2	
							−1	−1/2	4/5	1/5	5/2
							−2	+1/2	1/5	−4/5	−5/2
									−2	−1/2	1

Look for row labeled $-1 \ +1/2$

$$|2 - 1\rangle \left|\frac{1}{2}\frac{1}{2}\right\rangle = \sqrt{\frac{2}{5}}\left|\frac{5}{2} - \frac{1}{2}\right\rangle - \sqrt{\frac{3}{5}}\left|\frac{3}{2} - \frac{1}{2}\right\rangle$$

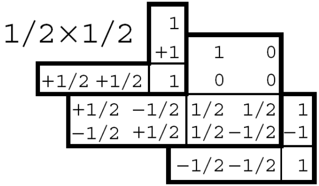
The probability to get $j = 5/2$ is $2/5$ and $j = 3/2$ is $3/5$.

Clebsch-Gordon Coefficients

Example 4.4

We know that 2 spin-1/2 states combine to either give spin-1 or spin-0. Find the explicit Clebsch-Gordon decomposition for these states.

$$
\begin{aligned}
|\tfrac{1}{2}\tfrac{1}{2}\rangle|\tfrac{1}{2}\tfrac{1}{2}\rangle &= |11\rangle \\
|\tfrac{1}{2}\tfrac{1}{2}\rangle|\tfrac{1}{2}-\tfrac{1}{2}\rangle &= \frac{1}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{2}}|00\rangle \\
|\tfrac{1}{2}-\tfrac{1}{2}\rangle|\tfrac{1}{2}\tfrac{1}{2}\rangle &= \frac{1}{\sqrt{2}}|10\rangle - \frac{1}{\sqrt{2}}|00\rangle \\
|\tfrac{1}{2}-\tfrac{1}{2}\rangle|\tfrac{1}{2}-\tfrac{1}{2}\rangle &= |1-1\rangle
\end{aligned}
$$



Read down the the rows

Thus the three spin 1 states (triplet) are:

$$
\begin{aligned}
|11\rangle &= |\frac{1}{2}\frac{1}{2}\rangle|\frac{1}{2}\frac{1}{2}\rangle \\
|10\rangle &= \frac{1}{\sqrt{2}}(|\frac{1}{2}\frac{1}{2}\rangle|\frac{1}{2}-\frac{1}{2}\rangle + |\frac{1}{2}-\frac{1}{2}\rangle|\frac{1}{2}\frac{1}{2}\rangle) \\
|1-1\rangle &= |\frac{1}{2}-\frac{1}{2}\rangle|\frac{1}{2}-\frac{1}{2}\rangle
\end{aligned}
$$

whereas the spin 0 state (singlet) is:

$$
|00\rangle = \frac{1}{\sqrt{2}}(|\frac{1}{2}\frac{1}{2}\rangle|\frac{1}{2}-\frac{1}{2}\rangle - |\frac{1}{2}-\frac{1}{2}\rangle|\frac{1}{2}\frac{1}{2}\rangle)
$$

Clebsch-Gordon Coefficients

1/2×1/2	1			
	+1	1	0	
+1/2 +1/2	1	0	0	
+1/2 −1/2		1/2	1/2	1
−1/2 +1/2		1/2	−1/2	−1
−1/2 −1/2				1

Read down the columns

$$
\begin{aligned}
|1 1\rangle &= |\tfrac{1}{2}\tfrac{1}{2}\rangle|\tfrac{1}{2}\tfrac{1}{2}\rangle \\
|1 0\rangle &= \frac{1}{\sqrt{2}}\left(|\tfrac{1}{2}\tfrac{1}{2}\rangle|\tfrac{1}{2}-\tfrac{1}{2}\rangle + |\tfrac{1}{2}-\tfrac{1}{2}\rangle|\tfrac{1}{2}\tfrac{1}{2}\rangle\right) \\
|1 -1\rangle &= |\tfrac{1}{2}-\tfrac{1}{2}\rangle|\tfrac{1}{2}-\tfrac{1}{2}\rangle \\
|0 0\rangle &= \frac{1}{\sqrt{2}}\left(|\tfrac{1}{2}\tfrac{1}{2}\rangle|\tfrac{1}{2}-\tfrac{1}{2}\rangle - |\tfrac{1}{2}-\tfrac{1}{2}\rangle|\tfrac{1}{2}\tfrac{1}{2}\rangle\right)
\end{aligned}
$$

How to read a CG table

- First, find the appropriate table for the j_1 and j_2 of interest.
- If it's m_1 and m_2 that are known and we want to find j, read the appropriate row across. Don't forget that all entries have an implicit square root. For example:

$$|\,1\,0\,\rangle\,|\,1/2\;\;1/2\,\rangle = \sqrt{2/3}\;|\,3/2\;\;1/2\,\rangle - \sqrt{1/3}\;|\,1/2\;\;1/2\,\rangle$$

- If it's j that is known and we want to find m_1 and m_2, read the appropriate column down. For example:

$$\begin{aligned} |\,3/2\;\;1/2\,\rangle &= \sqrt{1/3}\;|\,1\,1\,\rangle\,|\,1/2\;\;-1/2\rangle \\ &\quad+\sqrt{2/3}\;|\,1\,0\,\rangle\,|\,1/2\;\;1/2\,\rangle \end{aligned}$$

Spin 1/2

The most important spin system is spin-1/2 (leptons, quarks, some baryons, etc.).
States are described as linear combination of 2-component spinors

$$\chi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \alpha\chi_+ + \beta\chi_-$$

In practice, we do not measure a mixture of states but rather either spin-up or spin-down

$$\chi_+ = \left|\frac{1}{2}\frac{1}{2}\right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \chi_- = \left|\frac{1}{2} - \frac{1}{2}\right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

In the general case

$$|\alpha|^2 + |\beta|^2 = 1$$

and α and β give the relative probabilities of the two states.
Spinors (two component objects) occupy intermediate position between scalars (one component) and vectors (three components).

Suppose we want to measure S_x or S_y on a particle in state

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

We know that the values must be $\pm 1/2\hbar$ but what are the probabilities of each?
Introduce the Pauli Matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and

$$\hat{S} = (\hbar/2)\sigma$$

Example 4.5

Suppose we measure S_x^2 on a particle in the state $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$. What values might we get and what is the probability of each?

The matrix representing the square of S_x^2 is simply

$$\begin{aligned} \hat{S}_x^2 &= \frac{\hbar^2}{4}\sigma_x^2 \\ \hat{S}_x^2 &= \frac{\hbar^2}{4}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

The allowed values are the eigenvalues of the matrix and

$$\frac{\hbar^2}{4}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \frac{\hbar^2}{4}\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

So, any spinor of that form is an eigenvector of S_x^2 and the eigenvalue is $\hbar^2/4$ in all cases.

Isospin

- The neutron (939 MeV) and the proton (938 MeV) have almost the same mass. $n-n$, $n-p$ & $p-p$ interactions are identical if EM interaction ignored.
- Heisenberg suggested n and p were actually two different states of the same particle, the nucleon.
- We distinguish the nucleons by saying that they have different isospin. Define nucleon spinor:

$$N = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

with

$$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- All we have done is “borrow” the whole apparatus of spin and apply it to the nucleon with a new name.

Isospin

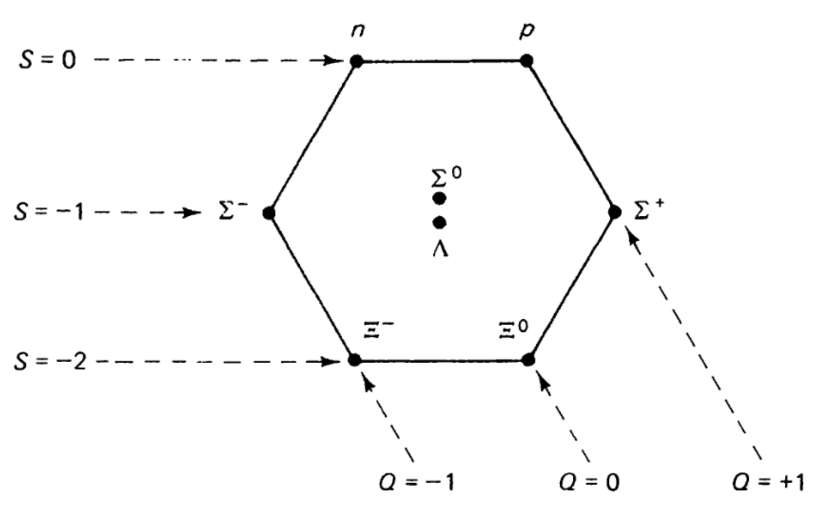
- The nucleon is isospin=1/2 and the third component has eigenvalues +1/2 (proton) and -1/2 (neutron)

$$p = \left|\frac{1}{2}\frac{1}{2}\right\rangle, \qquad n = \left|\frac{1}{2} - \frac{1}{2}\right\rangle$$

- *I* is a vector in “isospin space”. The strong interaction is invariant under rotations in this space.
- This symmetry means that isospin is conserved in processes mediated by the strong interaction.
- We call this an internal symmetry, because it has nothing to do with space and time, but rather with the relations between different particles.

Isospin Assignments

Baryon Octet:



Isospin Assignments

For any hadrons made up of *u* and *d* quarks, construct isospin multiplets:

$$p = \left|\frac{1}{2}\frac{1}{2}\right\rangle, \qquad n = \left|\frac{1}{2} - \frac{1}{2}\right\rangle$$

For $I = 0$ we have

$$\Lambda = |00\rangle$$

For $I = 1$

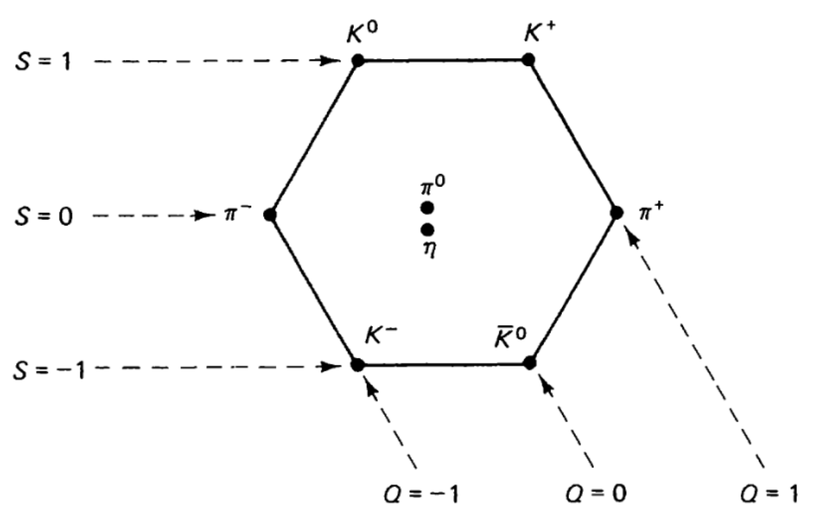
$$\Sigma^+ = |11\rangle\,, \qquad \Sigma^0 = |10\rangle\,, \qquad \Sigma^- = |1-1\rangle$$

For $I = 1/2$

$$\Xi^- = \left|\frac{1}{2} - \frac{1}{2}\right\rangle, \qquad \Xi^0 = \left|\frac{1}{2}\frac{1}{2}\right\rangle$$

Isospin Assignments

Meson Octet:



Isospin Assignments

$$K^{+} = \left|\frac{1}{2}\frac{1}{2}\right\rangle, \qquad K^{0} = \left|\frac{1}{2} - \frac{1}{2}\right\rangle$$

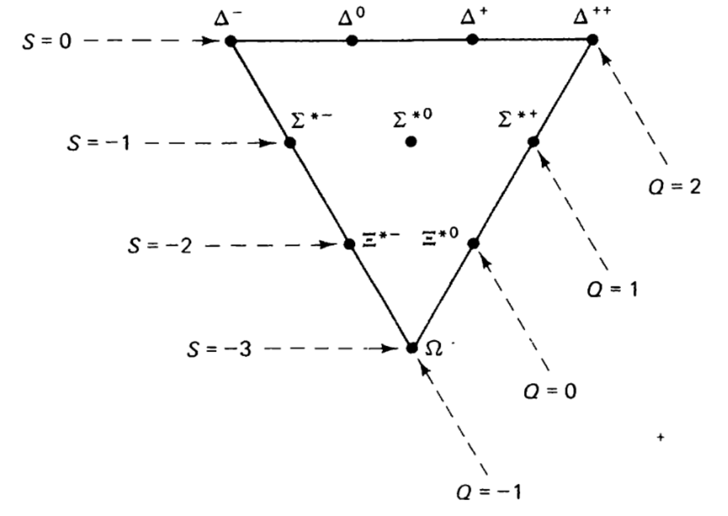
$$\eta = |00\rangle$$

$$\pi^{+} = |11\rangle, \qquad \pi^{0} = |10\rangle, \qquad \pi^{-} = |1-1\rangle$$

$$K^{-} = \left|\frac{1}{2} - \frac{1}{2}\right\rangle, \qquad \bar{K}^{0} = \left|\frac{1}{2}\frac{1}{2}\right\rangle$$

Isospin Assignments

Baryon Decuplet:



Isospin Assignments

For $I = 3/2$ we get the Δ's

$$\Delta^{++} = \left|\frac{3}{2}\frac{3}{2}\right\rangle, \quad \Delta^{+} = \left|\frac{3}{2}\frac{1}{2}\right\rangle, \quad \Delta^{0} = \left|\frac{3}{2} - \frac{1}{2}\right\rangle, \quad \Delta^{-} = \left|\frac{3}{2} - \frac{3}{2}\right\rangle$$

For $I = 0$ we have

$$\Omega^{-} = |00\rangle$$

Isospin of Deuteron

- The deuteron is composed of a proton and a neutron, which in the language of isospin couple together as:

$$\left|\frac{1}{2}\frac{1}{2}\right\rangle\left|\frac{1}{2}-\frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |00\rangle)$$

- Is the deuteron the isotriplet $|10\rangle$ state or isosinglet $|00\rangle$ state?
- If the deuteron is $|10\rangle$, then the other two members of the isotriplet should also exist with similar properties. BUT... neither *nn* nor *pp* states exist as stable nuclei, hence we conclude that the deuteron is an isosinglet.

Isospin: *nucleon – nucleon* scattering

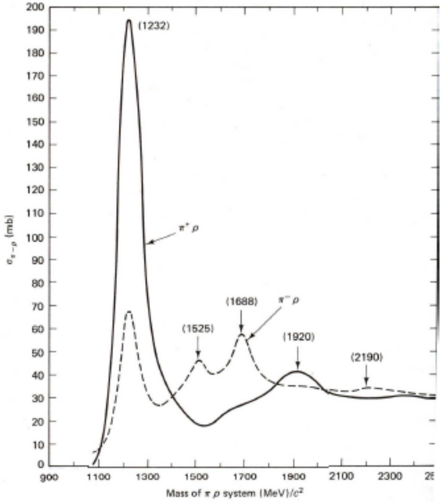
Isospin invariance has implications on *nucleon – nucleon* scattering. Consider

$$
\begin{aligned}
(a)\quad \underbrace{p+p}_{|11\rangle} \quad &\rightarrow \quad \underbrace{d+\pi^+}_{|11\rangle} \\
(b)\quad \underbrace{p+n}_{\frac{1}{\sqrt{2}}(|10\rangle+|00\rangle)} \quad &\rightarrow \quad \underbrace{d+\pi^0}_{|10\rangle} \\
(c)\quad \underbrace{n+n}_{|1-1\rangle} \quad &\rightarrow \quad \underbrace{d+\pi^-}_{|1-1\rangle}
\end{aligned}
$$

The $I=0$ component of the second reaction does not contribute and

$$\sigma_a : \sigma_b : \sigma_c = 2 : 1 : 2$$

Isospin: π – *nucleon* scattering



If only $I = 3/2$ component contributes then:

$$\frac{\sigma_{tot}(\pi^+ + p)}{\sigma_{tot}(\pi_- + p)} = 3$$

$\Delta(1232)$ is an $I = \frac{3}{2}$ resonance.
Data and theory look good together

Isospin can help us predict relative cross sections without having to know anything about the absolute cross sections.

Flavor Symmetries

- Isospin is an $SU(2)$ flavor symmetry whose origin lies in the near degeneracy of the masses of the u and d quarks.
- The s quark is also somewhat lighter than most hadrons, an $SU(3)$ flavor symmetry will also provide a useful description of hadron physics. This $SU(3)_F$ flavor symmetry is what got Murray Gell-Mann his nobel prize.
- Flavor symmetries, arising from the near-degeneracy of the light quark masses, provide us with free information about the strong interaction.

Discrete Symmetries

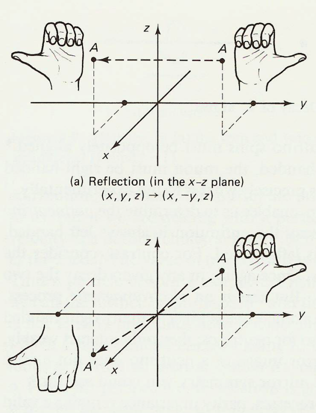
Parity

Parity

A parity transformation inverts every spatial coordinate

$$P(t, \boldsymbol{x}) = (t, -\boldsymbol{x})$$

- It is a reflection + a 180° rotation
- $P^2 = I$, therefore the eigenvalues are ± 1



Parity Eigenvalues

- Consider an ordinary vector $\boldsymbol{v}$. By the definition of parity $P(v) = -v$.
- Now let's contruct a scalar from $\boldsymbol{v}$: $s = v \cdot v$

$$P(\boldsymbol{v} \cdot \boldsymbol{v}) = (-\boldsymbol{v}) \cdot (-\boldsymbol{v}) = \boldsymbol{v} \cdot \boldsymbol{v} = +s$$

- Now try the cross product of two vectors: $\boldsymbol{a} = \boldsymbol{v} \times \boldsymbol{w}$

$$P(\boldsymbol{v} \times \boldsymbol{w}) = (-v) \times (-w) = \boldsymbol{v} \times \boldsymbol{w} = +\boldsymbol{a}$$

- Finally, we can form a scalar from $\boldsymbol{a}$ and $\boldsymbol{v}$: $p = \boldsymbol{a} \cdot \boldsymbol{v}$

$$P(\boldsymbol{a} \cdot \boldsymbol{v}) = (+a) \cdot (-v) = -\boldsymbol{a} \cdot \boldsymbol{v} = -p$$

Types of Scalars and Vectors

Scalar	$P(s) = +s$
Pseudoscalar	$P(p) = -p$
Vector	$P(v) = -v$
Pseudovector	$P(a) = +a$

Note: pseudovectors are also known as axial vectors

Parity in Physical Systems

- Parity is a multiplicative quantum number, like all discrete symmetries.
  Continuous symmetries lead to additive quantum numbers
- 2-body systems have parity $p_A p_B (-1)^l$ where l is the orbital angular momentum eigenvalue.
- Particles and anti-particles have opposite parity.
  Bound states like positronium ($e^+ e^-$) and mesons ($q\bar{q}$) have an overall parity of $(-1)^{(l+1)}$.
- Photons have a parity of (-1), and this underlies the $\Delta l = \pm 1$ selection rule in atomic transitions.

Parity Example: $u\bar{u}$ mesons

- By convention:
  u quarks have spin 1/2 and $+$ parity
  and
  $\bar{u}$ quarks have spin 1/2 and – parity
- Parity of a $u\bar{u}$ meson is then

$$P = p_u p_{\bar{u}} (-1)^l$$

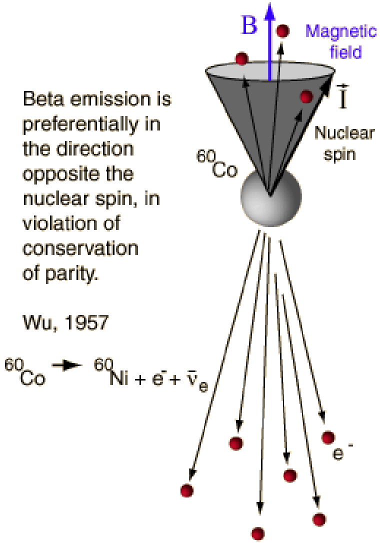
- The intrinsic spin of this meson is 0 or 1 but can have any orbital angular momentum:

S	L	J^p	particle	transformation
0	0	0^-	π^0	pseudoscalar
1	0	1^-	ρ^0	vector
0	1	1^+	$b_1(1235)$	pseudovector

Parity in the SM

- The particle physics community had just assumed that parity was a “good” symmetry of nature. In fact, perfectly good theories had been discarded because they would not respect this symmtery.
- In 1956 Yang and Lee noticed that nobody had ever shown that the weak interaction respects this symmetry and proposed an experiment which was then performed by Madame Wu

Parity in the SM



The nuclear spin is an axial vector (even under parity) and the electron momentum is a vector (odd under parity). The relative orientation of the two changes under parity.
Any assymetry in the electron distribution relative to the spin (***B***) direction violates parity.

The electron was emitted in the same direction independent of the nuclear spin.

Parity violation is the signature of weak interaction.

Helicity and Handedness

- If we have to choose one component of the angular momentum (the z component) we might as well choose the direction to be the direction of motion of the particle.
- Define helicity as $h = m_s/s$,
  spin-1/2 particles have either
  $h = -1$ (left-handed)
  or
  $h = +1$ (right-handed).

(a) Right-handed (b) Left-handed

Fig. 4.9 Helicity. In (a) the spin and velocity are parallel (helicity +1); In (b) they are antiparallel (helicity −1).

Parity Violation in π Decay

- Consider the weak process $\pi^+ \to \mu^+ + \nu_\mu$. Since the π is spin-0 and the outgoing particles are back-to-back in the CM frame the spins must cancel.
- Experiments show that every μ^+ is left-handed which makes every ν_μ left handed.
- Similarly in π^- decay, both the μ^- and the $\bar{\nu}_\mu$ always emerge right handed.

Fig. 4.10 Decay of π^- at rest.

- If parity were conserved by the weak interaction we would expect an equal number of right and left-handed pairs with equal probability (just as we observe with $\pi^0 \to \gamma\gamma$ decay through EM interaction.)

Chirality

We will learn later that the weak interaction selects only chiral-left particles and chiral-right anti-particles. That is where the L comes from in

$$SU(2)_L$$

Chirality is Lorentz invariant, helicity is not. However, for massless particles chirality reduces to helicity and we see only helicity-left particles.

Neutrinos are Left-Handed

Assuming massless neutrinos,
all neutinos are left-handed
and
all anti-neutrinos are right-handed.

Charge Conjugation

- The charge conjugation operator turns a particle into its antiparticle
$$C\,|p\rangle = |\overline{p}\rangle$$
- *C* reverses **every** internal quantum number (ie. charge, lepton/baryon number, strangeness, etc.)
- $C^2 = I$ implies the only allowed eigenvalues of *C* are ± 1
- Unlike parity, very few particles are *C* eigenstates. Only those particles which are their own antiparticles (π^0, η, γ, etc.) are *C* eigenstates.

Using Charge Conjugation

- The photon has $C = -1$
- $f\bar{f}$ bound states have $C = (-1)^{l+s}$
- Charge conjugation is respected by both the strong and EM interactions
- The π^0 ($l = s = 0 \rightarrow C = +1$) can decay into 2γ but not 3γ

$$\begin{aligned} C\,|n\gamma\rangle &= (-1)^n\,|\gamma\rangle \\ C\,|\pi^0\rangle &= |\pi^0\rangle \end{aligned}$$

G-Parity

- Most particles are not C-eigen states, hence C-symmetry is of limited use.
- The C-operator converts π^+ to π^-.
- These two have isospin assignments:

$$\pi^+ = |11\rangle\,, \qquad \pi^- = |1-1\rangle$$

- A π rotation in isospin gives:

$$|11\rangle = e^{i\pi I_2}\,|1-1\rangle$$

- The charged pions are eigenstates under the G-parity operator

$$G = Ce^{i\pi I_2}$$

- G-parity is mainly used to examine decays of pions (which have G=-1):

$$G\,|n\pi\rangle = (-1)^n\,|n\pi\rangle$$

G-Parity of a few light mesons

Particle	J^P	I	G	Decay	width (MeV)
$\rho(770)$	1^-	1	+1	2π	150
$\omega(783)$	1^-	0	-1	3π	8.5
$\phi(1020)$	1^-	0	-1	3π	4.3
$f(1270)$	2^+	0	+1	2π	185

Remember - G-Parity involves isospin, so it only tells us about *strong* decay selection rules.

For example, the $\rho(770)$ has G = 1 which means it should only decay to an even number of pions. Experimentally we find that:

$$\rho \to \pi\pi \qquad 100\%$$

$$\rho \to \pi\pi\pi \qquad < 1.2\text{x}10^{-4}\%$$

Conserved Quantum Numbers

Quantity		Strong	EM	Weak
Charge	Q	✓	✓	✓
Baryon Number	B	✓	✓	✓
Lepton Number	L	✓	✓	✓
Strangeness	S	✓	✓	✗
Isospin	I	✓	✗	✗
	I_3	✓	✓	✗
Parity	P	✓	✓	✗
Charge Conjugation	C	✓	✓	✗

CP Symmetry

- Reconsidering the decay

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

recall that the ν_μ is always left-handed

- Under charge conjugation we have

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

but the $\bar{\nu}_\mu$ is still left-handed and that does not exist in nature.

- However, if we combine C and P then everything is OK again. Perhaps this is the reflection symmetry we should have been looking for all along??
- Nope, wrong again....

CP Violation in the Kaon Sector

Neutral kaons K^0 and $\overline{K}^0$ with quark assignments $d\bar{s}$ and $s\bar{d}$ respectively.

These particles can mix via a second-order weak interaction since the weak interaction does not conserve quark flavour:

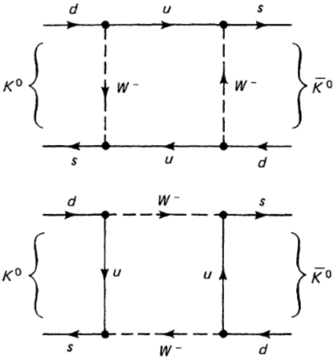


Fig. 4.12 Feynman diagrams contributing to $K^0 \rightleftharpoons \bar{K}^0$. (There are others, including those with one or both u quarks replaced by c or t.)

Kaon Quantum Numbers

- Both K^0 and $\bar{K}^0$ are pseudoscalar mesons with $P = -1$
- And K^0 and $\bar{K}^0$ are a particle-antiparticle pair so

$$
\begin{aligned}
C\,|K^0\rangle &= |\bar{K}^0\rangle \\
C\,|\bar{K}^0\rangle &= |K^0\rangle
\end{aligned}
$$

- As a result under CP we have

$$
\begin{aligned}
CP\,|K^0\rangle &= -\,|\bar{K}^0\rangle \\
CP\,|\bar{K}^0\rangle &= -\,|K^0\rangle
\end{aligned}
$$

Kaon CP Eigenstates

- What we actually measure is not K^0 or $\bar{K}^0$ but some linear combination of the two.
- Defining:

$$\begin{aligned} |K_1\rangle &= (|K^0\rangle - |\bar{K}^0\rangle)/\sqrt{2} \\ |K_2\rangle &= (|K^0\rangle + |\bar{K}^0\rangle)/\sqrt{2} \end{aligned}$$

giving

$$\begin{aligned} CP\,|K_1\rangle &= +|K_1\rangle \\ CP\,|K_2\rangle &= -|K_2\rangle \end{aligned}$$

- If CP is conserved then $|K_1\rangle$ can only decay to 2π ($CP = +1$) and $|K_2\rangle$ can only decay to 3π ($CP = -1$)
- Based on phase-space considerations, $|K_1\rangle$ should have a much shorter lifetime than $|K_2\rangle$

Kaon Decay Eigenstates

- We observe K_1 and K_2 with lifetimes of 0.9×10^{-10}s and 0.5×10^{-7}s
- K^0 and $\bar{K}^0$ are mass eigenstates and are each others antiparticles
- K_1 and K_2 are approximately CP eigenstates (have different masses) and are not antiparticles
- The fact that they were not exactly CP eigenstates was astounding.

The Experiment

- Start with a beam of kaons. Can use an arbitrarily long beam to get arbitrarily pure K_2. The K_1's should decay in a few cm.
- K_2 decays only to 3π, never to 2π if CP is a good symmetry.
- Cronin and Fitch made a 57 foot long beamline in 1964 and observed 45 2π decays in 22700. Unmistakable evidence of CP violation.
- In other words, K^0 and $\bar{K}^0$ are the particle-antiparticle pair, K_1 and K_2 are linear combinations of those states and the long-lived kaon we observed is not pure K_2 but rather

$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}(|K_2\rangle + \epsilon\,|K_1\rangle)$$

Kaon Eigenstates

- K^0 and the $\overline{K}^0$ are mass eigenstates and are each others antiparticles
- K_1 and the K_2 are CP eigenstates
  $\left|K^0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K_1\right\rangle + \left|K_2\right\rangle\right)$ $\qquad$ $\left|\overline{K}^0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K_1\right\rangle - \left|K_2\right\rangle\right)$
- K_S^0 and the K_L^0 are the observed states and are nearly identical to the CP eigenstates (K_S^0 and K_L^0 are not antiparticles)
  $\left|K_L^0\right\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}\left(\left|K_2\right\rangle + \epsilon\left|K_1\right\rangle\right)$ $\qquad$ $\left|K_S^0\right\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}\left(\left|K_2\right\rangle - \epsilon\left|K_1\right\rangle\right)$

Other Tests of CP

- You can also see *CP* violation in kaons by looking at

$$\begin{aligned} K_L &\rightarrow \pi^+ + e^- + \bar{\nu}_e \\ K_L &\rightarrow \pi^- + e^+ + \nu_e \end{aligned}$$

- Notice that *CP* changes one of these into the other so they should be equally probable.
- K_L is observed to decay more often into a positron by a fractional amount 3.3×10^{-3} and *CP* is violated again.
- We can now distinguish between matter and antimatter. Positive charge is the charge carried by the lepton preferentially produced in the decay of the long-lived neutral K-meson. Explanation for matter-antimatter assymmetry???
- *CP* violation has been found in *B*'s and is being searched for in charmed mesons.

Why study CP violation?

- Sakarov pointed out that it is possible to start from a matter-antimatter symmetric universe and end up in one that is asymmetric.
  This requires that there be some process (or processes) that violates the CP symmetry.
- The SM does not predict CP violation (it can accomodate no CPV or CPV). However, the SM provides only one source of CP violation (CKM phase angle) which is only possible if there are more than 3 quark generations.
- The currently observed SM CPV (using K and B mesons) is too small to explain the matter-dominated universe.
- There is another possible source of CPV within the SM in the neutrino sector

Time Invariance

- Time reveral symmetry reverses the time component

$$T(t, \boldsymbol{x}) = (-t, \boldsymbol{x})$$

- We expect to observe T violation but there is no experimental evidence of it so far.
  Experimentally, one tries to measure the rate of a reaction in both directions $A + B \to C + D$ but this is not so easy.

CPT

The reason we expect to see T violation is that we think that CPT is a good symmetry (it is required by QFT).
If CP is allowed to be violated in some process then we must be allowed to violate T too to make CPT invariant.

CPT Theorem

The combination CPT is always conserved in any local quantum field theory.
CPT violation in essentially synonymous with a violation of Lorentz invariance.

CPT symmetry mandates that particles and antiparticles must have certain identical properties, such as the same mass, lifetime, charge, and magnetic moment.

More Symmetries - Lepton Number

- There are 3 lepton numbers: L_e, L_μ, L_τ with (for example)

$$\begin{aligned} L_e &= +1 \text{ for } e^-, \nu_e \\ L_e &= -1 \text{ for } e^+, \bar{\nu}_e \end{aligned}$$

  These are additive quantum numbers.
- They are conserved in EM and weak interactions. So, $\mu^+ \to e^+\gamma$ is forbidden (for example).
- People are actively putting new limits on lepton flavour violation in BaBar with processes like $\tau^+ \to \mu^+\gamma$ ($BR < 10^{-8}$)
- Neutrino oscillations imply that lepton number is violated (at a very small level).
  Lepton Number conservation is not exact.

More Symmetries - Baryon Number

- We can associate with each baryon (3-quark bound state) a quantum number called Baryon number which has a similar definition

$$\begin{aligned} B &= +1 \; \textit{for } p, n, ... \\ B &= -1 \; \textit{for antiprotons, antineutrons}, ... \end{aligned}$$

- It appears to be conserved in all interactions. if it werent we (i.e. protons) might decay into to lighter particles. Hence the stability of the proton. Lifetime of proton is $> 10^{31} - 10^{33}$ years.